Axisymmetric Flow with Swirl

There are a number of important contexts in which understanding of axisymmetric flow with swirl is important. Important examples are the dynamics of rotating meteorological phenomena such as tornadoes, hurricanes, etc. and the rotating flows associated with turbomachines and cyclone separators. For ease of reference we repeat from section (Bgfa) the basic equations governing axisymmetric potential flow with swirl:

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{Bgfc1}$$

$$\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_\theta u_r}{r} = 0 \tag{Bgfc2}$$

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} \tag{Bgfc3}$$

The middle equation (Bgfc2) provides an evolution equation for the swirl velocity, u_θ, namely

$$\frac{D(r\ u_\theta)}{Dt} = 0 \tag{Bgfc4}$$

and shows that the quantity ru_θ remains constant along any streamline. Also the pressure can be eliminated from the top and bottom equations, (Bgfc1) and (Bgfc3), in a very similar way that was done for flows without swirl. In the present case that elimination again leads, as expected, to the appropriate form of the vorticity transport equation for this type of flow, namely,

$$\frac{D(r\omega_\theta)}{Dt} = \frac{\partial u_\theta^2}{\partial z} \tag{Bgfc5}$$

where, as defined in equation (Bgfa4), the only non-zero vorticity component in this type of flow, ω_θ, is given by

$$r\omega_\theta = r\left\{\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right\} = -\frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} \tag{Bgfc6}$$